Life Earnings and Rural-Urban Migration

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This paper is a theoretical study of rural-urban migration—urbanization—as it has occurred in many low-income economies in the postwar period. This process is viewed as a transfer of labor from a traditional, land-intensive technology to a human capital–intensive technology with an unending potential for growth. The model emphasizes the role of cities as places in which new immigrants can accumulate the skills required by modern production technologies.

I. Introduction

The origins of the modern economic world can be seen, in part, as a transition from a traditional agricultural society to a society of sustained growth in opportunities, of human and physical capital accumulation. In the countries in which the Industrial Revolution is well under way, this transition is complete. The share of Britain's population living in rural areas had already fallen to 50 percent by 1850 and reached 11 percent by 1998. The share of the workforce in agriculture declined

The original version of this paper was prepared for the May 19, 2001, Symposium in Honor of Sherwin Rosen. Writing it has been in part an attempt to prolong the pleasure of talking about economics with Sherwin, an activity that was an important part of my life for many years. Many participants in the symposium offered useful comments and suggestions. In particular, Kevin Murphy showed me how to exploit the linearity of the model to streamline and strengthen the results. Robert Shimer suggested a natural alternative model that accounts for some of the observations that motivated me. Robert Hall conjectured the efficient allocation of the model and provided an unsettling *reductio ad absurdum* to the technology I assumed. I am also grateful to V. V. Chari, Zvi Eckstein, Ivar Ekeland, D. Gale Johnson, Patrick Kehoe, Steven Levitt, Ellen McGrattan, Jordan Rappaport, Nancy Stokey, Robert Tamura, Yoram Weiss, and participants at seminars at the University of Minnesota, the Federal Reserve Bank of Minneapolis, Vanderbilt University, and Arizona State University for helpful discussions. Finally, I thank Alejandro Rodriguez and Mikhail Golosov for their very able assistance.

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TABLE 1

Country	Share of Rural Population (%) 1950 (1)	2000 (2)	Population in Largest City (Millions of People) 1950 (3)	2000 (4)	Share of Largest City in Urban Population (%) 1950 (5)	2000 (6)
Argentina	35	10	5.3	12.6	47	38
Brazil	75	19	2.8	17.8	15	13
Egypt	68	55	2.5	10.6	39	34
India	84	72	2.9	18.1	5	6
Mexico	58	26	3.1	18.1	27	25
Philippines	74	41	1.6	10.9	29	24
South Korea	79	18	1.1	9.9	25	26
Thailand	90	78	1.4	7.3	66	55

Source.—World Bank, *World Development Indicators*, various issues.

from 21 percent in 1851 to 7 percent by 1911 to 2 percent in 1995. In the United States, the fraction of the labor force in agriculture fell from 79 percent in 1820 to 40 percent in 1900, to 23 percent in 1930, and then to 3.4 percent in 1980.

In the last half of the twentieth century, many developing economies have undergone transitions from rural to urban much more rapidly than the leaders did. In South Korea, the fraction of the labor force in agriculture fell from 63 percent in 1963 to 22 percent in 1987. The share of South Korean gross national product originating in agriculture declined from 39 percent to 12 percent over the same 24-year period.[1] Between 1950 and 2000, the rural share of Mexico's population fell from 58 to 26 percent, and Mexico City grew from 3 million to about 18 million people (see cols. 1–4 for Mexico in table 1). As this table shows, similar transitions occurred between 1950 and 2000 in Brazil, the Philippines, and South Korea. In Argentina, much of this transition had already occurred by 1950. In India, Thailand, and Egypt, it is clearly under way, but just beginning. In the countries listed in table 1, two economies can be seen side by side: The modern business centers of Manila and São Paulo coexist with rural poverty at something like preindustrial levels.

The share of agriculture in production and employment is declining everywhere. Agricultural goods generally have income elasticities below one, and for this reason alone, agricultural production should grow more slowly than production in general. In the wealthy countries, technological change has proceeded more rapidly in agriculture than elsewhere. These forces, studied in Shin (1990) and elsewhere, are of course

[1] Figures for the United States and South Korea are taken from Shin (1990).

reflected in table 1. They are not the focus of the present study. My interest in this paper lies rather in the role migration out of agriculture plays in a society's transition from an economy based on traditional agriculture to an industrialized, perpetually growing economy. This transition is an irreversible process that every industrializing society undergoes once and only once. Economies in the midst of this transition—Brazil, Mexico, the Philippines, and South Korea in table 1—are middle-income economies that for a few decades exhibit *much* more rapid urbanization than either much poorer or much richer economies do. This process, too, is reflected in table 1.

A successful theory of the urbanization process must have several distinct features. First, it must describe a migration out of traditional agriculture that continues until this sector disappears. The agriculture that remains in the advanced economies is a part of the modern economy, characterized by sustained productivity growth just as much as manufacturing and services are. Second, this process takes many decades to complete, with a long period of coexistence of the traditional and modern economies. Third, the process involves income equalization among those who have migrated. All of our families began in traditional agriculture, but it is hard to tell those of us who made the transition 100 years ago from those who migrated 300 years ago. There are no second-generation busboys and manicurists in American cities.[2]

Some other aspects of the transitions illustrated in the table are puzzling. Many of the new immigrants to these cities seem to be worse off than they were in the rural areas they came from. They live as squatters, in shanty towns. They have no regular jobs. Measured unemployment in Mexico City is 25 percent. In Manila it is 17 percent. Why can these immigrants not bid themselves into high-wage city jobs? Why do they keep coming? Todaro (1969) and Harris and Todaro (1970) addressed some of these questions in two celebrated papers. They model migration to the city as entering a lottery in which the winners get a high-wage city job and the losers are unemployed. There is an equilibrium in which the expected wage for a migrant equals the rural wage. But what keeps the high city wage high in these circumstances? What prevents the lottery losers from going back home?

In this paper, I propose a different way of addressing these same

[2] Livas and Krugman (1992) and Ades and Glaeser (1995) see the growth of the largest cities in these transitional economies as a new phenomenon, possibly because of a concentration of rent-seeking activities. But as cols. 5 and 6 of table 1 illustrate, the urbanization in these countries represents a growth of cities in general at the expense of the countryside, not growth in the largest cities relative to smaller ones. Mexico City's share in the rapidly growing urban population of Mexico is very stable. The same can be said for the largest cities in the seven other countries in table 1. Eaton and Eckstein (1997) find stability in the largest city's share in total urban population for Japan over the period 1925–85 and France from 1876 to 1990.

questions. I follow Harris and Todaro in treating urban-rural wage differentials as equalizing, but in the model I develop, high city wages reflect a high skill level, and these jobs are not available to low-skilled immigrants from the country. The immigrants come, as in the model of Eaton and Eckstein (1997), because cities are good places to accumulate human capital, and it is the return to this activity that equals the rural wage in equilibrium.[3]

In all the models developed in this paper, individual families are viewed as infinitely lived dynasties in which all ages are always represented. They solve time allocation problems of the type studied by Rosen (1976): A fixed time endowment is allocated between working at a wage that is dictated by one's current skill level and accumulating human capital so as to increase future earnings. In Section II, I work out the equilibrium allocations that result from this behavior in a traditional agricultural economy and in a modern, human capital–based economy, both considered first in isolation. In Section III the possibility of migrating from the agricultural to the urban economy is then studied in a setting in which the return from investing in skills accrues entirely to the person making the investment.

In the theory of Section III, the share of gross domestic product produced in the traditional economy goes to zero at a realistic rate, but migration occurs all at once, at the first opportunity. The rest of the paper addresses this problem. In Section IV, an external effect of human capital is added to the learning technology: Time invested in human capital accumulation has a higher return in high–human capital environments. In this situation, a decision not to migrate early on can be reversed later on because the city becomes an ever more attractive destination. Equilibrium is defined for this case in Section IV, some asymptotic properties are developed in Section V, and numerical results are presented and discussed in Section VI. These three sections are the main contribution of the paper.

Section VII develops an alternate model that also accounts for a gradual process of migration. In this model, only the initial migrants accumulate human capital, but there are complementary jobs for unskilled labor in the city as well. As the early migrants become more skilled, more unskilled workers can find attractive city jobs. I argue that although this model captures interesting aspects of reality, its prediction of a permanent, ever-widening gulf between skilled and unskilled city workers is not borne out. Section VIII concludes with a discussion of the relation of the theory to observations and some speculations about future research directions.

[3] This idea is also reflected in the introductory discussion in Todaro (1969). Glaeser and Mare (2001) provide evidence consistent with it.

II. Two Polar Cases

Throughout the paper I consider two-sector (rural and urban) economies, with a fixed total population of identical households, viewed as infinitely lived dynasties. The main focus will be on the forces affecting the flow of people from farm to city, but I begin by setting notation and describing resource allocation for specialized all-rural and all-urban economies.

Every family has preferences

$$\int_0^\infty e^{-\rho t} U(c(t))\, dt \tag{1}$$

over paths $c(t)$, $t \geq 0$, of a single, nonstorable consumption good. Assume that

$$U(c) = \frac{1}{1-\sigma} c^{1-\sigma}.$$

Each household has one unit of nonleisure time, supplied inelastically to income-directed activities: working for wages and accumulating human capital.

In the agricultural economy, land and labor are combined to produce the consumption good. Take the total population to be one, so that total and per capita magnitudes have the same symbols. Normalize the total amount of land at one as well and write $F(x(t))$ for farm production, where $x(t)$ is farm employment. The function F is taken to be Cobb-Douglas: $F(x) = Ax^{\alpha}$. Human capital is assumed to have no effect on productivity in agriculture, so no time there is devoted to human capital accumulation. In an economy in which the entire workforce is employed in agriculture, then, the competitive equilibrium real wage is $w = F'(1)$, equilibrium consumption is $c = F(1)$, land rents are $F(1) - F'(1)$, and the interest rate is constant at $r = \rho$.

The assumptions just stated—in particular the absence of technological change and the constant level of income—are aspects of what I called a "traditional agricultural" economy in the Introduction. They are a reasonably accurate description of the economies of pre-industrial Europe and of large parts of the poor countries of the world today. They do not describe agriculture in the twentieth-century United States. In fact, the process of industrialization feeds back on the practice of agriculture, incorporating this sector into the modern economy of sustained technological change and capital accumulation. By the time the agricultural workforce is 3 percent of the total, the employment share of *traditional* agriculture is zero. The models constructed in this paper do not address this aspect of the transition.

Next let us consider a second economy, identified as urban or city, in which there is a linear, labor-only production technology. Under this technology, a worker with skill level $h(t)$ who devotes $u(t)$ units of time to goods production produces $u(t)h(t)$ units of the consumption good. Initially, I assume that human capital accumulation depends only on the household's own actions, according to

$$\frac{dh(t)}{dt} = \delta h(t)[1 - u(t)], \tag{2}$$

where $u(t)$ is the fraction of time spent producing the consumption good. As in Rosen's (1976) analysis, the time $1 - u(t)$ is to be thought of as including *all* knowledge-improving activities, useful experience on and off the job, as well as schooling.[4]

Under the assumption of perfect capital markets and given the assumed absence of leisure in the utility function, every household will allocate its time so as to maximize the present value of its wage income. In the urban sector, the linear production technology fixes the real wage at a constant, which I take to be unity. Let the path of instantaneous interest rates faced by households be $r(t)$, $t \geq 0$. Then each household will choose the functions $u(t)$ and $h(t)$, $t \geq 0$, so as to maximize

$$\int_0^\infty \exp\left[-\int_0^t r(s)ds\right] h(t)u(t)dt \tag{3}$$

subject to (2) and the constraint $u(t) \in [0, 1]$.

The first-order condition for an interior maximum, $u(t) \in (0, 1)$, is

$$h(t) = \delta \int_t^\infty \exp\left[-\int_t^\tau r(s)ds\right] h(\tau)u(\tau)d\tau, \tag{4}$$

where the left side is the opportunity cost of devoting one unit of time to human capital accumulation, and the right side is the discounted return from this investment. Both the objective function (3) and the constraint (2) are linear in the decision variables $u(t)$, so if (4) holds at any date t, the household is indifferent among all $u(t) \in [0, 1]$ at that date.

Under the accumulation technology (2), $h(t)$ and $h(\tau)$ are related by

$$h(\tau) = h(t) \exp\left\{\delta \int_t^\tau [1 - u(s)]ds\right\}.$$

[4] Rosen (1976) assumes that the rate of growth in human capital is linear in the stock $h(t)$, as in my (2), but not in effort $u(t)$.

Substituting into (4) and canceling $h(t)$ gives

$$1 = \delta \int_t^{\infty} \exp\left[-\int_t^{\tau} r(s)ds\right] \exp\left\{\delta \int_t^{\tau} [1 - u(s)]ds\right\} u(\tau)d\tau. \tag{5}$$

Differentiating both sides of (5) with respect to time yields

$$r(t) = \delta, \tag{6}$$

which is to say that the equilibrium interest rate must equal δ whenever people are both producing goods and accumulating human capital. This is just a consequence of the linearity of the capital accumulation technology (2) in *both* $h(t)$ and $u(t)$: if such a linear investment technology is used in equilibrium, it will dictate the return $r(t) = \delta$ on *all* investments.

The household will have some non–human wealth, too, since land is a factor of production. If we call a the sum of the value of land and the human wealth defined in (3), we can complete the statement of the household's problem: Choose a consumption path $c(t)$, $t \geq 0$, so as to maximize (1) subject to

$$\int_0^{\infty} \exp\left[-\int_0^{t} r(s)ds\right] c(t)dt \leq a.$$

The first-order condition for this problem can be written

$$U'(c(t)) = U'(c(0)) \exp\left[\rho t - \int_0^{t} r(s)ds\right].$$

Under the particular preferences assumed here, differentiating this condition with respect to time yields the familiar formula

$$\frac{1}{c(t)}\frac{dc(t)}{dt} = \frac{r(t) - \rho}{\sigma}. \tag{7}$$

In an economy in which the urban technology is the only one available, the competitive equilibrium will be an Ak model of endogenous growth. From (6), the interest rate is constant at $r = \delta$. Then (2) and (7) imply

$$\frac{1}{c(t)}\frac{dc(t)}{dt} = \frac{1}{h(t)}\frac{dh(t)}{dt} = \delta[1 - u(t)] = \frac{\delta - \rho}{\sigma}.$$

Thus $u(t)$ is constant at the value

$$v = 1 - \frac{\delta - \rho}{\delta\sigma}. \tag{8}$$

Since $u(t) \in [0, 1]$, (8) requires that the model's parameters satisfy

$$\delta\sigma \geq \delta - \rho \geq 0. \tag{9}$$

We impose (9) for the remainder of the paper.

III. A Model of Transition

With the two extreme cases of the last section as background, I turn to the study of an economy's transition from one extreme to the other. The assumptions on markets, preferences, and the two technologies from the last section remain in force. The maximum problems solved by households are the same, except that every household is also free to work in either the rural or the urban sector.

Consider an economy in which everyone is initially in the rural sector, and everyone has available the common human capital level h_0 whenever he migrates to the city. This capital is assumed to be useless in agricultural production. In urban production, it enables a single full-time worker to produce h_0 units of the consumption good. At each date, every worker has to decide whether to live in the country or to move to the city (or to move in the reverse direction), and, if he is in the city, how to divide his time between goods production and human capital accumulation. I assume perfect capital markets and identical initial wealth (landholdings) for all households, so that the consumption level implied by these two labor allocation decisions will be distributed equally over families. Each family wishes to maximize the utility (1).[5]

The assumption that capital markets are complete, used throughout the paper, requires thinking of the representative agent as a dynastic family that serves all its generational cohorts as a capital market. I think that this abstraction is particularly suitable to a study of migration. The initial outlay migration requires is very modest, and early migrants from a family can and do repay loans by helping out later migrants as well as by remittances. Within such a family, migrants can be selected for family wealth maximization, much as they would be in a well-functioning, external capital market (see, e.g., Chen, Chiang, and Leung 2003).

An individual household in this economy has three choices to make at each date: how to allocate income between consumption and savings, how to allocate labor time between city and farm, and, for urban members, how to allocate time between learning and producing. The first-order condition for the first decision is (7). As was shown in Section II, the first-order condition for the third decision implies a value for the

[5] A model very close to the model described in this section, and with similar properties, is developed in Glomm (1992).

equilibrium interest rate, $r(t) = \delta$. Combining these facts yields one equilibrium condition,

$$\frac{1}{c(t)}\frac{dc(t)}{dt} = \frac{\delta - \rho}{\sigma}, \tag{10}$$

carried over from the Ak economy.

The first-order condition for the decision on work location is simply that the present value of working forever at either location must be equal in the city and the farm. Farm earnings are $F'(x(t))$. City earnings for a new migrant are h_0 times the fraction of time $u(t)$ spent working. Earnings at any later date τ are

$$h(\tau)u(\tau) = h_0 \exp\left\{\delta \int_t^\tau [1 - u(s)]ds\right\} u(\tau)$$

if he chooses the time allocation program $u(s)$ between the migration date t and τ. The interest rate is constant at δ. Hence the equality of farm and city present values is expressed by

$$\int_t^\infty \exp[-\delta(\tau - t)]F'(x(\tau))d\tau =$$

$$h_0 \int_t^\infty \exp[-\delta(\tau - t)] \exp\left\{\delta \int_t^\tau [1 - u(s)]ds\right\} u(\tau)d\tau. \tag{11}$$

Notice that there are no moving costs in (11). An individual might costlessly move back and forth many times, but if he ever moves back from city to farm, he is wasting any urban human capital he has accumulated. Such a move must reflect a mistake, and in the deterministic context I am using here, such mistakes will not occur.

Condition (11) can be simplified by using an integration by parts to evaluate the integral on the right. Provided that

$$\lim_{t\to\infty} \int_0^t u(s)ds = +\infty$$

(i.e., provided that urban production time remains bounded away from zero), we have

$$h_0 \int_t^\infty \exp[-\delta(\tau - t)] \exp\left\{\delta \int_t^\tau [1 - u(s)]ds\right\} u(\tau)d\tau = \frac{h_0}{\delta}. \tag{12}$$

Condition (12) reflects the fact that with this linear technology for both production and learning, all ways of allocating one's time yield equal value. In particular, then, working full-time and never learning yields earnings h_0 and, discounting at the rate δ, yields the present value h_0/δ. Equation (12) says that no behavior can yield higher value than this particular strategy does.

Combining (11) and (12) and differentiating with respect to time yields

$$F'(x(t)) = h_0. \tag{13}$$

Since $F'(x)$ is strictly decreasing, (13) implies that the equilibrium value of $x(t)$ will be constant over time at the value $x_0 = (F')^{-1}(h_0)$. All the rural-urban migration that ever takes place will take place at date $t = 0$. In this case, all urban producers will be identical, and there is no loss of generality in assuming that they all have a common time allocation $u(t)$ and human capital path $h(t)$ that satisfy (2).

With a constant allocation of the workforce between farm and city, the goods market–clearing condition is

$$c(t) = F(x_0) + (1 - x_0)h(t)u(t). \tag{14}$$

Equations (14) and (10) together imply

$$u(t) = \frac{c(0)}{(1 - x_0)h(t)}\exp\left(\frac{\delta - \rho}{\sigma}t\right) - \frac{F(x_0)}{(1 - x_0)h(t)}.$$

Inserting this time allocation behavior into (2) yields

$$\frac{dh(t)}{dt} = \delta h(t) - \delta\frac{c(0)}{1 - x_0}\exp\left(\frac{\delta - \rho}{\sigma}t\right) + \delta\frac{F(x_0)}{1 - x_0}. \tag{15}$$

The initial value $h(0) = h_0$ is given, as is $x_0 = (F')^{-1}(h_0)$. Thus (15) defines a one-parameter family of human capital paths, indexed by the initial value $c(0)$ of consumption. The next result states that exactly one of these paths behaves asymptotically as in the Ak model of Section II.

Proposition 1. If $\delta - \rho \geq 0$ and $\sigma\delta - \delta + \rho > 0$, there is exactly one value $c(0)$ such that the corresponding solution $h(t)$ to (15) satisfies

$$\lim_{t\to\infty}\frac{1}{h(t)}\frac{dh(t)}{dt} = \frac{\delta - \rho}{\sigma}.$$

The proof is given in the Appendix.

Figure 1 illustrates the behavior of the time allocation and the share of production that occurs in cities for the model of this section, for a particular set of parameter values. At all dates on this figure, half of the labor force is in the city and half in the country. Also at all dates, consumption is growing at $(\delta - \rho)/\sigma = .02$ (under the parameter values

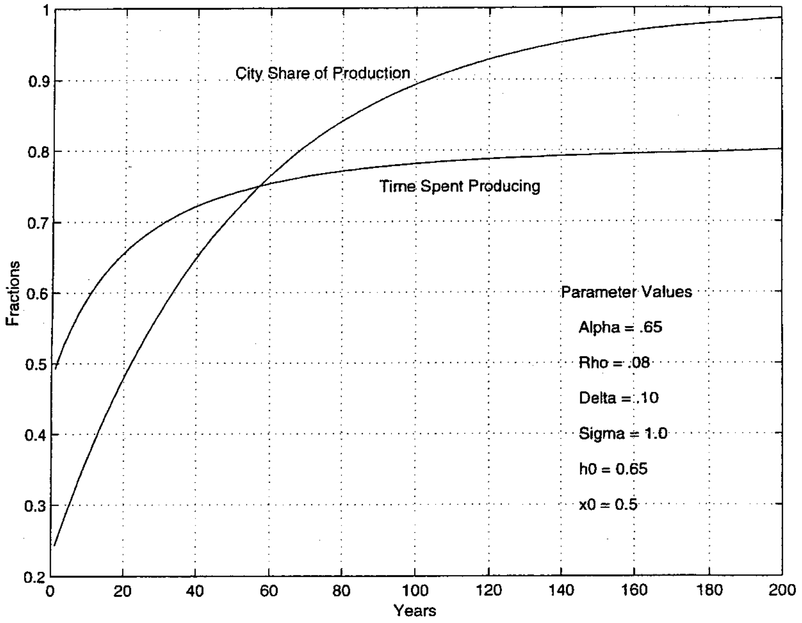

FIG. 1.—Allocation of time and production

listed on the figure) per year. Eventually, the urban share of production must go to one, in which case

$$u(t) \rightarrow v = 1 - \frac{\delta - \rho}{\delta\sigma} = 0.8. \tag{16}$$

These facts essentially dictate the starting value $u(0)$ and the rate at which the convergence in (16) will occur.

IV. Transition with External Effects

What is missing in this picture? In the economy shown in figure 1, people migrate from a land-using agricultural technology to a human capital–intensive technology that I have called urban, and in the transition the growth rate of production increases from zero to a sustained, positive level. The share of agricultural production in GDP goes from one to zero. These are all features that the theory is designed to capture.

But the migration that occurs in this model occurs once and for all, leaving a constant fraction of the labor force permanently agricultural. Moreover, terminology aside, there is nothing really urban about the growth technology. Under this technology, city producers have no connection with one another except through the capital market: As producers, they operate as so many independent Robinson Crusoes.

We know that production can be given a spatial dimension by postulating a production externality that makes any individual more productive if other productive people are nearby (as in Glomm [1992]). Pursuing this idea would make it easier to think of the human capital–intensive technology as "urban," but by itself it would not alter the basic dynamics of the model. Instead, I make use here of a formulation proposed and analyzed by Eaton and Eckstein (1997) in which an externality affects the technology for accumulating human capital rather than the technology for producing goods.

This modification will require notation that lets us describe an urban population in which different people have different levels of human capital. Accordingly, let $h(s, t)$ denote the human capital at t of a person who migrated to the city at date $s \leq t$. Note that we still require that everyone in the migration cohort s behave identically. We also continue to assume that entry-level human capital is $h(t, t) = h_0$.

Now let $H(t)$ denote the highest skill level that any worker in the economy has attained at date t, and assume that the learning technology is given by

$$\frac{\partial h(s, t)}{\partial t} = \delta\left[\frac{H(t)}{h(s, t)}\right]^{\theta} h(s, t)[1 - u(s, t)]. \tag{17}$$

As compared to the technology (2), (17) magnifies the effect δ by an increasing function of the gap between one's own human capital $h(s, t)$ and the human capital of the leader, $H(t)$. (As will be shown in a moment, no one with human capital less than the level $H(t)$ of the leaders will produce anything, so $H(t)$ can also be interpreted as the *average* skill level of urban producers.)

Setting $\theta = 0$ in (17) gives the case studied in the last section. Eaton and Eckstein study the case $\theta = 1$ for balanced paths in an economy with many cities but no rural sector. In this context, as they say, the added generality obtained by leaving θ free in (17) does not add new qualitative possibilities. For my purposes, however, the ability to vary the externality parameter θ will be helpful.

With the learning technology described by (17), migration to the city will become increasingly attractive over time, as those who have migrated earlier accumulate better and better skills. This will require that wages on the farm also rise continuously, which will in turn require that the farm workforce fall continuously. We need notation to let us discuss these possibilities.

At any date t, let $x(t)$ be the number of people who are employed in agriculture. Their total production is $F(x(t))$. As in Section III, if x_0 is defined by $F'(x_0) = h_0$, then $x(0) = x_0$ people will immediately move to the city. This is simply a switch from the technology $F(x)$ to the superior technology

$$G(x) = \int_0^x \max\,[h_0,\ F'(y)]dy,$$

for using unskilled labor, as in Hansen and Prescott (2002), having nothing to do with human capital accumulation. After this initial movement, I assume that $x(t)$ is continuously differentiable and decreasing, so that $-x'(t)$ is the rate at which people migrate to the city at date t.

I adopt the symmetry assumption that all of the initial $1 - x_0$ migrants will allocate their time in the same way. Thus they will be the highest-skilled leaders, with human capital paths denoted $H(t) \equiv h(0,\ t)$, determined by their time allocation paths $u(t)$ and (17). City production $y(t)$ will be the sum of goods produced by the leaders and those produced by subsequent migrants:

$$y(t) = (1 - x_0)H(t)u(t) - \int_0^t x'(s)h(s,\ t)u(s,\ t)ds. \tag{18}$$

Total production includes agricultural production as well:

$$c(t) = F(x(t)) + y(t). \tag{19}$$

Under (17), workers with different skill levels face different returns to investing in human capital: the lower one's own $h(s,\ t)$ is relative to the leaders' $H(t)$, the easier it is to attain a given percentage growth rate. For this reason, it will not be possible to capture the information in the state $h(s,\ t)$ in a single, one-dimensional variable such as the stock $h(t)$ of Section III. As we shall see, the Eaton-Eckstein model offers a different simplifying principle.

As in Section III, we shall seek an interior equilibrium in which at every date there is *some* worker who divides his time between producing and learning. In this case, as before, the equilibrium interest rate is always equal to δ. Then the present value of earnings for a city worker who migrates at date s and chooses the time allocation path $u(s,\ t)$ and the human capital path $h(s,\ t)$, $s \le t$, is

$$\int_s^\infty e^{-\delta(t-s)}h(s,\ t)u(s,\ t)dt. \tag{20}$$

These choices are constrained by (17), where $H(t)$ is viewed as a given function of time and $h(s,\ t)$ has the initial value $h(s,\ s) = h_0$. Each worker wants to choose his time allocation $u(s,\ t)$, $t \ge 0$, so as to maximize (20) subject to (17).

To characterize the solution to this problem, we need the correct price to value an increment to human capital. Let the continuous-time Bellman equation for household h be

$$\delta w(h, H) = \max_{u\in[0,1]} [hu + w_h(h, H)\delta H^{\theta}h^{1-\theta}(1-u)] + w_H(h, H)H'(t),$$

so the first-order condition can be written

$$1 - w_h(h, H)\delta H^{\theta}h^{-\theta} \leq 0 \tag{21}$$

(with equality if $u > 0$). For the leaders, $h = H$, and we know that the leaders choose $u > 0$. Thus

$$1 = w_h(H, H)\delta,$$

reproducing the conclusion from the last section that the marginal value of capital is $1/\delta$. Everyone else has the option of using an increment Δh to human capital to produce forever at the rate Δh and obtain the value $\Delta h/\delta$. Thus $w_h(h, H) \geq 1/\delta$ for all $h \leq H$. Then for $h < H$,

$$1 - w_h(h, H)\delta H^{\theta}h^{-\theta} \leq 1 - H^{\theta}h^{-\theta} < 1, \tag{22}$$

and (21) and (22) imply $u = 0$.

This is essentially Eaton and Eckstein's proof that the human capital leaders will be the *only* urban producers in equilibrium: Everyone else will specialize in investing in human capital. This means that all new migrants specialize and will continue to specialize until they catch up and become leaders themselves. The external effect creates two classes of city dwellers: producers and full-time learners. This fact lets us streamline the description of production provided in (18)–(20) as follows.

Let $z(t)$ be the number of leaders at date t: the number of workers who have attained the skill level $H(t)$ of the leaders and are now producing goods as well as accumulating human capital. We have just shown that only these workers produce goods, so in place of (19) and (20), we write

$$c(t) = F(x(t)) + z(t)H(t)u(t). \tag{23}$$

The human capital of these leaders evolves according to

$$\frac{dH(t)}{dt} = \delta H(t)[1 - u(t)], \tag{24}$$

repeating (2). The human capital for someone who migrates at date s evolves for $t \geq s$ according to

$$\frac{\partial h(s,\ t)}{\partial t} = \delta H(t)^{\theta} h(s,\ t)^{1-\theta}, \tag{25}$$

repeating (17) but with $u(s,\ t) = 0$.

Comparing (24) and (25), one can see that for every migration cohort s there will be a catch-up date $T(s)$, say, at which $h(s,\ t) = H(t)$ first holds. Given a human capital path $H(t)$ for the leaders, (25) is an ordinary differential equation in $h(s,\ \cdot)$ for each fixed s. Provided that $\theta > 0$, the solution to this equation with the initial condition $h(s,\ s) = h_0$ is

$$h(s,\ t) = \left[h_0^{\theta} + \theta\delta \int_s^t H(u)^{\theta} du\right]^{1/\theta}. \tag{26}$$

The catch-up date $T(s)$ for a date s migrant is thus given implicitly by

$$h_0^{\theta} + \theta\delta \int_s^{T(s)} H(u)^{\theta} du = [H(T(s))]^{\theta}. \tag{27}$$

In terms of the catch-up time function T, the number of urban producers $z(t)$ must satisfy the time delay formula

$$z(T(t)) = 1 - x(t). \tag{28}$$

The economics of the migration decision requires that the present value of earnings in the rural sector, as given by the left side of (11), should equal the present value of earnings in the city. As we saw in Section III, the present value of earnings to a city resident who is both producing and learning at every date will be $H(t)/\delta$. But the migrant at t will not begin producing until date $T(t)$. Equilibrium thus requires that

$$\int_t^{\infty} e^{-\delta(s-t)} F'(x(s)) ds \geq e^{-\delta[T(t)-t]} \frac{H(T(t))}{\delta} \tag{29}$$

(with equality if $x'(t) < 0$) hold for all t. (Under the Cobb-Douglas technology, the rural technology will never be entirely abandoned, so there is no loss of generality in comparing the present value of a migrant's earnings to the value of the earnings of someone who remains *permanently* rural.)

Over a time interval on which (29) holds with strict inequality, $x'(t) = 0$ and farm employment $x(t)$ will be constant. On an interval on which (29) holds with equality, differentiating both sides of (29) with respect to time and applying (24) yields

$$F'(x(t)) = e^{-\delta[T(t)-t]}H(T(t))u(T(t))T'(t).$$

Differentiating both sides of (27) then gives the formula

$$u(T(s))T'(s) = \left[\frac{H(s)}{H(T(s))}\right]^{\theta}.$$

Combining these facts implies that

$$F'(x(t)) = e^{-\delta[T(t)-t]}[H(T(t))]^{1-\theta}[H(t)]^{\theta} \tag{30}$$

holds whenever migration is positive.

To sum up the discussion to this point, an *equilibrium* for the economy of this section will be a collection of continuous, nonnegative functions $\{c(t), u(t), H(t), z(t), T(t)\}$ on $\mathbf{R}_+$; a nonnegative function $x(t)$ on $\mathbf{R}_+$ with a bounded, continuous, nonpositive derivative $x'(t)$; a function $h(s, t)$ on $\mathbf{R}_+ \times \mathbf{R}_+$; and numbers c_0 and x_0 such that (i) $u(t) \le 1$ for all t; (ii) $c_0 > 0$ and

$$c(t) = c_0 \exp\left(\frac{\delta - \rho}{\sigma} t\right) \tag{31}$$

for all t; (iii) $x(0) = x_0 \in [0, 1]$ and

$$F'(x_0) = h_0; \tag{32}$$

(iv) $H(t) = h(0, t)$ for all t; and (v) (18), (23), (24), and (26)–(29) are satisfied for all $t \ge 0$.

An iterative procedure can be used to construct an equilibrium. We begin by taking a farm workforce function $x(t)$ as a given, requiring it to have an initial value $x(0) = x_0$ satisfying (32) and have a continuous derivative. Then we simplify by eliminating some equations and unknowns, as follows. Combining (23), (24), and (31), we eliminate the variables $c(t)$ and $u(t)$ to obtain

$$\frac{dH(t)}{dt} = \delta H(t) - \frac{\delta}{z(t)}\left[c_0 \exp\left(\frac{\delta - \rho}{\sigma} t\right) - F(x(t))\right]. \tag{33}$$

Except for the variable $z(t)$, (33) is the same ordinary differential equation that describes equilibrium behavior in Section III. To describe the feedback from $H(t)$ to $z(t)$, it is useful to use the inverse function S of the catch-up time function T, writing

$$h_0^\theta + \theta\delta \int_{S(t)}^{t} H(u)^\theta du = H(t)^\theta \tag{34}$$

and

$$z(t) = 1 - x(S(t)) \tag{35}$$

in place of (27) and (28).

When a continuously differentiable function $x(t)$, with $x(0) = x_0$, is taken as given, equations (33)–(35) form a dynamic system in the variables $H(t)$, $z(t)$, and $S(t)$. This system is "backward looking," and though it is not quite an ordinary differential equation, it can be analyzed using the standard methods for such equations. The result we need is the following proposition.

PROPOSITION 2. Suppose $\theta \in (0, 1)$. Suppose that $x_0 \in (0, 1)$, $x(0) = x_0$, and $x'(t)$ is continuous on $t > 0$. Then for any initial values h_0 and c_0, there is a unique set of continuously differentiable functions H, S, and z on $\mathbf{R}_+$ that satisfy (33)–(35).

The proofs of propositions 2 and 3 are supplied in the Appendix.

Proposition 2 ensures that there is a one-parameter family of solutions, indexed by the initial consumption level c_0. As in Section III, only members of this family for which $u(t) \to v$ are interesting to us.

PROPOSITION 3. Suppose $\theta \in (0, 1)$, $\delta - \rho \geq 0$, and $\sigma\delta - \delta + \rho \geq 0$. Then for any $x_0 \in (0, 1)$, continuously differentiable, nonincreasing $x(t)$, there is at least one value c_0 such that if H, S, and z satisfy (33)–(35), $H(t)$ satisfies

$$\lim_{t\to\infty} \frac{1}{H(t)} \frac{dH(t)}{dt} = \frac{\delta - \rho}{\sigma}. \tag{36}$$

If the initial consumption level c_0 consistent with (36) is unique for all choices of the function $x(t)$, propositions 2 and 3 would together define a mapping from the set of continuously differentiable, nonincreasing functions $x(t)$ into the set of continuously differentiable functions H, S, and z on $\mathbf{R}_+$. I have not been able to show that this is the case. I have proceeded computationally as though it were, however, using a shooting algorithm to calculate paths H, S, and z that are consistent with a given migration function x. This algorithm effectively utilizes the information in the time allocation problem of urban workers and the market-clearing conditions for goods and labor markets.

I then use the information in the migration decision problem to close the system, mapping the H, S, and z so calculated into a new migration function x. This mapping is given by the marginal condition for the migration decision (29). On any interval on which migration $-x'(t)$ is positive, equation (30) will hold. Define $\hat{x}(t)$ as the solution to (30):

$$F'(\hat{x}(t)) = e^{-\delta[T(t)-t]}[H(T(t))]^{1-\theta}[H(t)]^{\theta}.$$

Since F is strictly concave, $\hat{x}(t)$ is uniquely determined, and so is the function Vx defined by

$$(Vx)(t) = \min[x_0, \hat{x}(t)].$$

Moreover, since

$$\int_t^{\infty} e^{-\delta(s-t)}F'((Vx)(s))ds \geq \int_t^{\infty} e^{-\delta(s-t)}F'(\hat{x}(t))ds,$$

this function $(Vx)(t)$ satisfies (29) for all t. The solutions reported below are all obtained by locating a fixed point x of this operator V.

V. Asymptotic Properties of Equilibrium

Some asymptotic features of the equilibrium defined in the last section can be worked out quite easily. I do so in this section. More detailed results, based on computational experiments, are presented in Section VI.

Let us begin with the properties of the catch-up time function $T(t)$, defined implicitly in (27) as the solution T to the equation

$$\Phi(T, t) \equiv h_0^{\theta} + \theta\delta\int_t^{T} H(u)^{\theta}du - H(T)^{\theta} = 0. \tag{37}$$

We have

$$\frac{\partial\Phi(T, t)}{\partial T} = \theta H(T)^{\theta}\left[\delta - \frac{H'(T)}{H(T)}\right] > 0,$$

so at most one T solves (37) for each t. When $t = 0$, $T = 0$ satisfies (37), so $T(0) = 0$. For any $t > 0$, $\Phi(t, t) \equiv h_0^{\theta} - H(t)^{\theta} < 0$ since $H(0) = h_0$ and H is strictly increasing. Since $\Phi(T, t) \to \infty$ as $T \to \infty$, a solution always exists and $T(t) > t$ if $t > 0$. The derivative of T is given by

$$T'(t) = \frac{H(t)^{\theta}}{H(T)^{\theta}}\left[1 - \frac{1}{\delta}\frac{H'(T)}{H(T)}\right]^{-1} > 0.$$

Asymptotically, the stock of the leaders' capital behaves like

$$H(t) \simeq K_1 \exp\left(\frac{\delta - \rho}{\sigma}t\right),$$

so (37) implies that

$$\theta\delta \int_t^T \exp\left(\theta \frac{\delta - \rho}{\sigma} u\right) du \simeq \exp\left(\theta \frac{\delta - \rho}{\sigma} T\right).$$

Evaluating the integral, canceling, and taking logs, we obtain the asymptotic approximation to the pipeline length, $T(t) - t$:

$$T(t) - t \simeq \frac{1}{\theta\delta}. \tag{38}$$

Reasonably, a small external effect—θ near zero—implies a long catch-up time.

Now apply (38) to the equality (30) to obtain

$$F'(x(t)) \simeq K_2 \exp\left(\frac{\delta - \rho}{\sigma} t\right). \tag{39}$$

Thus the marginal product of farm labor eventually goes to infinity at about the rate of growth of production. With a Cobb-Douglas technology (and many others), this implies that farm labor goes to zero. Notice that neither this conclusion nor the asymptotic rate of convergence depends on the size of the externality parameter θ. Of course, the constant K_2 does depend on θ!

VI. Computational Experiments

As shown in Section V, if the externality parameter θ takes on any positive value, the farm labor force will ultimately converge to zero. We are also interested in the nature of this transition: the speed with which it takes place and the way resources are allocated as it occurs. For these questions, we need to calibrate the model and calculate solutions numerically.

The agricultural technology is assumed to be Cobb-Douglas: $F(x) = Ax^{\alpha}$. I use Johnson's (1948) estimate of about .35 for the share of land in agricultural income to set $\alpha = .65$. This entails identifying the labor input in the model with a composite labor-plus-capital input in reality. Per capita income in the advanced countries grows at about .02 annually. The net (of depreciation) return on physical capital is around .10. The coefficient of risk aversion is on the order of one (log utility) or two. Using $\sigma = 1$, $r = .10$, and $\delta(1 - u) = .02$, we obtain the estimates $\rho = .08$ and $u = 0.8$. These estimates, used to construct figure 1, are also used in all the computations reported below.

The parameters A and h_0 depend on the units of output and labor input. Their magnitudes will be treated arbitrarily. In the calculations

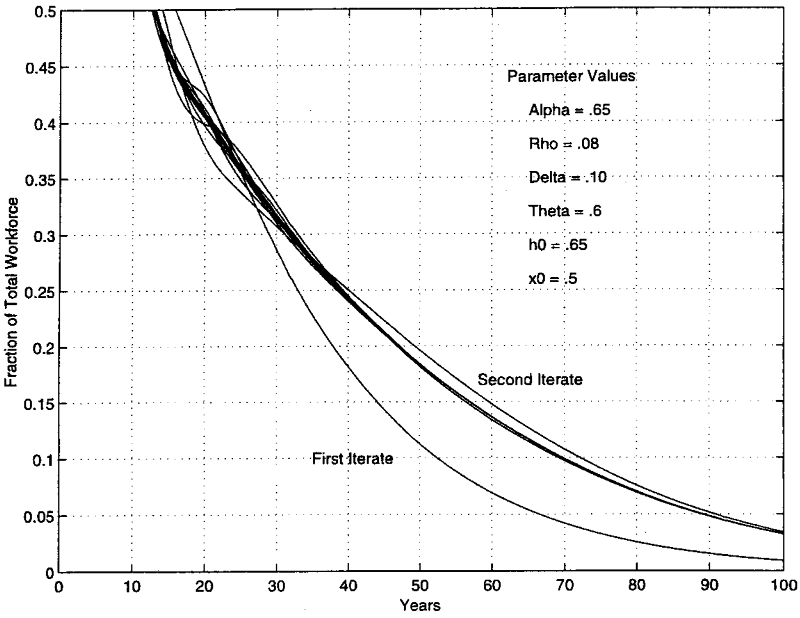

FIG. 2.—Farm employment, 30 successive approximations

to be reported, A is set equal to .785 and h_0 is chosen so that $F'(.5) = h_0$ (so that $x_0 = .5$).

Solutions were calculated by applying the operator V on farm employment paths $x(t)$ defined in Section IV. In all cases, the function $x^0(t) \equiv x_0$ was used to initiate the computation. In every case, the function $x^1(t) = (Vx^0)(t)$ satisfied $x^1(0) = x_0$ and was nonincreasing. When $\theta > 0$, it satisfied $x^1(t) \to 0$ as $t \to \infty$. These qualitative properties were satisfied for all the iterates $x^n(t) = (Vx^{n-1})(t)$.

Figure 2 shows the first 30 successive approximations obtained from the initial path $x(t) = x_0$ when the externality parameter takes the value $\theta = .6$. One can see from the figure that the operator generating the sequence of migration paths is not monotone, so the fact that the calculated equilibrium lies between extreme paths does not guarantee uniqueness, even within this range. In a model in which an external effect plays a key role, uniqueness is not just a technicality: The model would seem to have the potential for the kind of multiple equilibrium possibilities shown in Murphy, Shleifer, and Vishny (1989) or Matsuyama (1991). But I did not discover such possibilities in my numerical experiments with this model.

Figure 3 shows how the equilibrium farm employment paths $x(t, \theta)$,

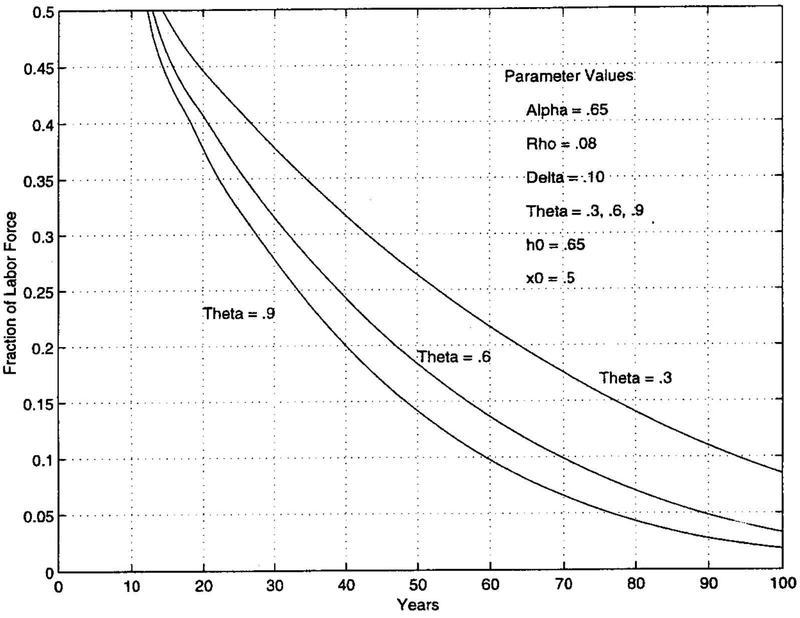

FIG. 3.—Farm employment for different θ values

say, vary as the externality parameter θ varies.[6] Recall that the case of $\theta = 0$ is shown in figure 1. There $x(t, 0) = x_0 = 0.5$ for all t. In general, all paths on figure 3 equal x_0 on some interval $[0, \tau(\theta)]$ and decline monotonically toward zero on $[\tau(\theta), \infty)$. For each fixed t, $x(t, \theta)$ is nonincreasing in θ. Whether $x(\cdot, \theta) \rightarrow x(\cdot, 0)$ as $\theta \rightarrow 0$ depends on the metric used (thus $\sup_t |x(t, \theta) - x(t, 0)| = 0.5$ for all $\theta > 0$), but everything that is interesting economically is continuous at $\theta = 0$.

The main lesson of figure 3 is that a larger external effect speeds up migration, in two ways. A lower θ increases the date $\tau(\theta)$ at which migration resumes. On the figure, $\tau(\theta)$ increases from 13 to 15 years as θ declines from .9 to .3. Then once migration resumes, farm population declines faster the higher θ is. The decrease in the farm workforce from 0.5 to 0.25, which occurs for every positive θ, takes 20 years if $\theta = .9$ and 38 years at $\theta = .3$.

Some features of the allocations associated with the middle equilibrium farm employment path in figure 3—the path associated with $\theta = .6$—are shown in figures 4, 5, and 6. Figure 4 shows the time al-

[6] These calculations and those reported below are based in a grid of 10 points per year over a period of 100 or (in some cases) 200 years. The operator V was iterated 100 times. The series $x(t)$ shown on fig. 3 was smooth, but the associated $u(t)$ and $z(t)$ series were not. Those shown in figs. 4 and 6 below were smoothed artificially.

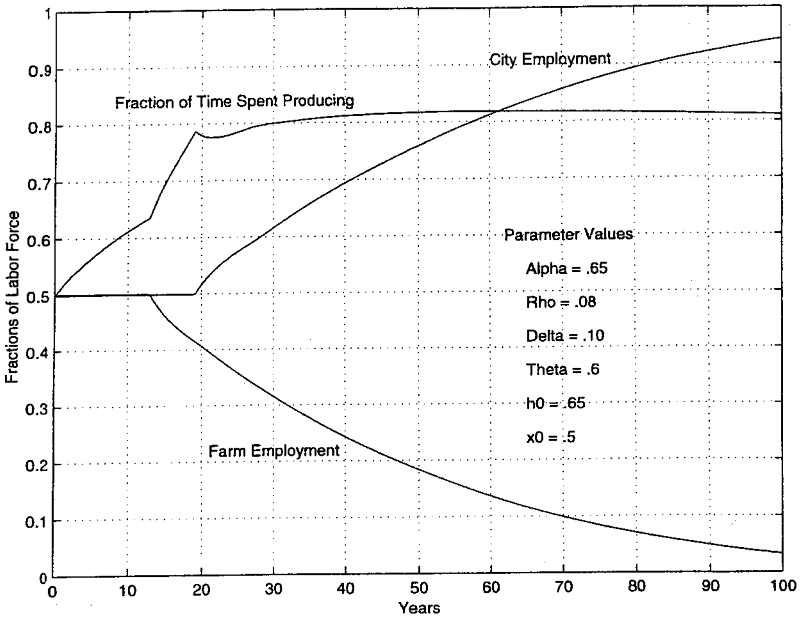

FIG. 4.—Employment and time allocations

location path $u(t)$ for urban workers, the path of farm employment, and the path $z(t)$ of city employment. There is an initial migration to the city of $1 - x_0$, and all these initial migrants are immediately employed in the city. No further migration occurs for 14 years, but the urban workers begin to accumulate human capital. Initially, half their time is so used; after 14 years, the fraction has declined to .37. At this point, human capital levels in the city have risen to the point at which migration from the farm again becomes attractive. This occurs because the external effect raises the return to investment by new migrants.

As $x(t)$ begins to decline, the time $u(t)$ that city workers spend producing increases sharply. Remember that consumption is growing at the constant annual rate of 2 percent at all dates in figure 4, so decreased farm production must be offset by increased urban production. The migrants from year 14 on spend their initial city years accumulating capital and producing nothing. Thus $z(t)$ remains at the level $1 - x_0$ until year 19, at which time the year 14 migrants begin to come on-line. At this point, $u(t)$ declines a little before resuming its approach to the steady-state level of $v = \rho/\delta = 0.8$.

The human capital paths associated with these curves are shown in figure 5. The capital path of the leaders is not quite exponential because $u(t)$ is not quite constant, but it is close. Also plotted are the human

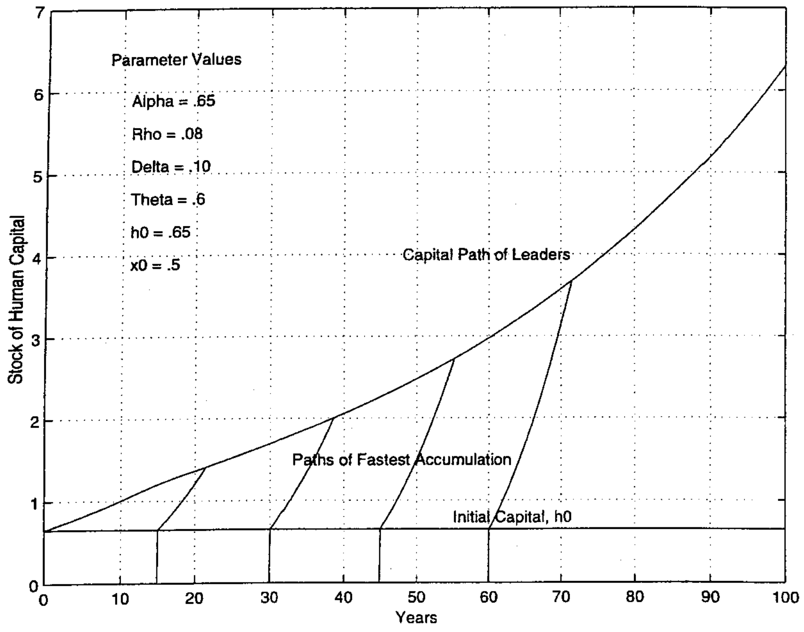

FIG. 5.—Human capital trajectories

capital paths of the cohorts migrating in years 15, 30, 45, and 60. These paths all begin at h_0 and climb rapidly toward the capital path of the leaders. These paths are steep in part because the migrants enjoy the beneficial externalities of those who have gone before, and this effect is amplified because the migrants spend full time learning, not just the fraction $u(t)$. When a migrant's path of accumulation reaches the leaders' path, the migrants begin to produce, adopting the time allocation $u(t)$ of the leaders, and no longer accumulate at the maximal rate.

The paths of city and farm employment shown on figure 4 do not sum to one because I have classed the migrants who are in the nonproducing, catch-up phase of their careers as "unemployed." The number so characterized is plotted as the middle curve on figure 6. Figure 6 shows three curves, corresponding to the three levels .3, .6, and .9 of the external effect that were used in constructing figure 3. For each curve, unemployment in this sense is zero until migration resumes, then rises to a peak in a few years, and then declines gradually to zero.

These curves are the contribution of this paper to the Harris-Todaro problem. One can see that with the high externality level of .9, unemployment begins to rise earliest, peaks at more than 15 percent of the urban workforce in about six years, and falls to 6 percent 30 years after that. At the other extreme, with $\theta = .3$, unemployment peaks at

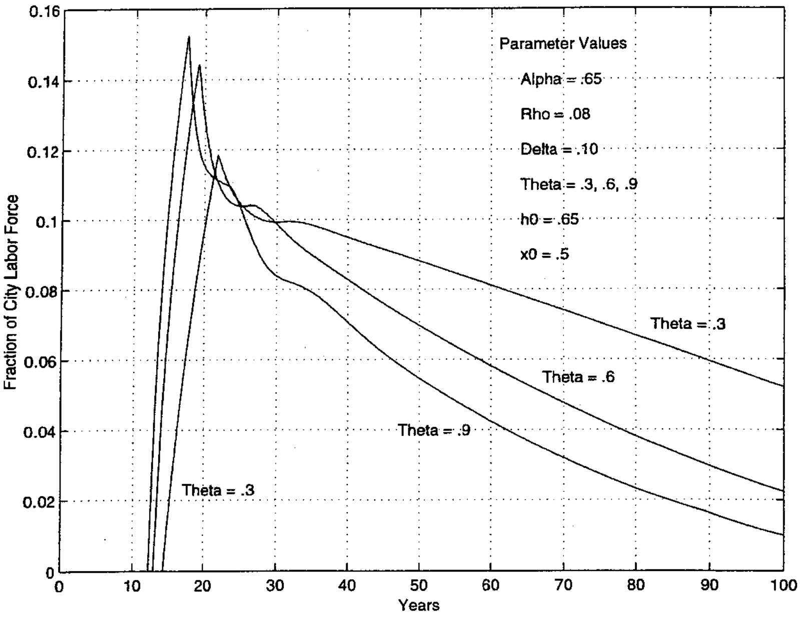

FIG. 6.—Urban unemployment rates, different θ values

less that 12 percent, but then takes nearly 70 years to decline to 6 percent.

Of course, this unemployment (if that is the right term for the series plotted in fig. 6) is entirely voluntary, a present-value-maximizing activity. As in the Harris and Todaro theory, people migrate to the city in full awareness of the economic consequences of doing so. In contrast to Harris and Todaro, though, jobs are available to everyone who wants one at wages suited to their individual skill levels.

VII. Transition with Two Urban Skill Levels

Another way of trying to account for a gradual pace of urbanization besides the externality model of Sections IV–VI would be to postulate a city technology in which skilled and unskilled labor are complementary factors of production.[7] Under such a technology, unskilled workers would be drawn to the city gradually to keep pace with the gradual

[7] This section follows up a suggestion by Rob Shimer. In a different but related context, Stokey (1996) uses a model with imperfectly substitutable labor skill levels to account for a gradual transition process. Rappaport (2000) uses slowly depreciating housing stocks to account for gradual migration in response to local productivity differences. Atkeson and Kehoe (1996) show that incomplete capital markets can have a major effect on transition dynamics.

growth of the human capital of skilled city workers. As in the model of Section III, people would divide themselves into skill-acquiring and permanently unskilled workers at date 0, and skilled workers would thereafter allocate their time so as to maximize the present value of their earnings, discounted at δ. Unskilled workers will be on the margin between working in the rural and urban sectors. Migration between technologies—here interpreted as migration between locations—will be drawn out through time, but migration across skill levels will not be. I develop this possibility in this section.

We shall continue to use the function $F(x(t))$ to describe production under the farm technology. In the city, now assume that $y(t)$ unskilled workers and $z(t)$ skilled workers produce

$$G(y(t),\ z(t)h(t)u(t))$$

units of the same good when the skilled workers have $h(t)$ units of human capital and devote the fraction $u(t)$ of their time to production. Assume that G has constant returns and denote its marginal product functions by G_u and G_s. The three worker categories must sum to one:

$$x(t) + y(t) + z(t) = 1. \tag{40}$$

At the initial date $t = 0$, all workers are identical. Anyone can work in the farm sector, earning the wage $F'(x(0))$, at an unskilled task in the city, earning the wage $G_u(y(0),\ z(0)h_0u(0))$, or at a skilled, city job, earning $G_s(y(0),\ z(0)h_0u(0))h_0$ per unit of time. Moreover, we know that the linearity of the learning technology implies that a worker choosing a skilled job will be indifferent between all time allocation choices $u(0)$. Labor market equilibrium thus implies

$$F'(x(0)) = G_u(y(0),\ z(0)h_0u(0)) = G_s(y(0),\ z(0)h_0u(0))h_0u(0). \tag{41}$$

Equations (40), at $t = 0$, and (41) are three equations in $x(0)$, $y(0)$, $z(0)$, and $u(0)$. A fourth equation is

$$c(0) = F(x(0)) + G(y(0),\ z(0)h_0u(0)). \tag{42}$$

As in earlier sections, the value of initial consumption must be chosen to ensure convergence to the Ak equilibrium.

The initial allocation of the labor force determines the *number* $z(0)$ of skilled workers for all time, though the effective labor these people provide, $z(0)h(t)u(t)$, evolves over time. The allocation of unskilled workers between city and farm jobs is then determined by

$$x(t) + y(t) + z(0) = 1 \tag{43}$$

and

$$F'(x(t)) = G_u(y(t),\ z(0)h(t)u(t)). \tag{44}$$

To compare the predictions of the model with two urban skill levels to the model with one urban skill in Section III, it is easiest to work with the Cobb-Douglas specifications $F(x) = Ax^{\alpha}$ and $G(y, z) = By^{\eta}z^{1-\eta}$. In this case, the wage of unskilled workers (wherever employed) relative to skilled workers is given by

$$\frac{w_u}{w_s} = \frac{G_u(y(t),\ z(0)h(t)u(t))}{G_s(y(t),\ z(0)h(t)u(t))h(t)u(t)} = \frac{\eta z(0)}{(1-\eta)y(t)}. \tag{45}$$

We know that $y(t) \to 1 - z(0)$ from above on an equilibrium path, since all unskilled workers eventually migrate. The formula (45) then implies that the skill differential increases over time, converging to a constant value.

In summary, the two-skill model of urban production implies that the fraction of the labor force in agriculture eventually goes to zero, just as the fraction of rural production does. This is a step toward realism from the transition dynamics described in Section III. Moreover, it is easy to see in reality the flows of low-skilled labor to cities to provide many of the services that high-skilled workers there are willing to pay for.

Thus the alternative model of this section captures something important, but I think that it misses some aspects of urbanization that are also important. New migrants to cities seem to undergo a learning process that reduces skill differentials over time. Cities do not remain rigidly stratified by skill differences: Mixing or catching up occurs that makes it hard, economically, to tell the more recent immigrants from the originals. These features are present in the model of Section IV, but not in the model with two skill levels. Moreover, the model of Sections IV–VI offers an interpretation of urban unemployment, similar to that offered by Harris and Todaro, that has no counterpart in the model of this section.

VIII. Conclusions and Possibilities

A useful theory of rural-urban migration—and hence of economic development—needs to be consistent with the gradual character of the urbanization process. Even in the rapidly growing economies of the postcolonial world, the passage from a 90 percent agricultural economy to one that is 90 percent urban occurs in a matter of decades. Since everyone has the option to migrate earlier rather than later, something must occur as time passes that makes the city a better and better destination.

One possibility, discussed in Section VII, is that the increasing skill levels of urban producers continuously raise the demand for complementary unskilled workers. But cities are much more than a source of jobs for unskilled people from the countryside: They are places in which people face new opportunities and accumulate new skills. As I have modeled these forces, they lead to an ever-widening skill differential between city and traditional agricultural workers that continuously draws new migrants. The higher the skill level in the city to which one moves, the more rapidly one's own skills accumulate and so the higher the return to the investment.

In figure 6, I plotted the number of migrants in the learning "pipeline," as a fraction of the urban labor force, against time, calling the series the urban unemployment rate. When the migration processes take off, as triggered when the urban skill level reaches a critical level, this rate rises rapidly to 10 or 15 percent and remains at a high level for many years (though not permanently). I was pleased that these figures are of the same order of magnitude as measured rates in Manila and Mexico City (though this identification is admittedly tenuous) and propose this as an alternative explanation to Harris and Todaro's for the apparent fact that many people leave low-wage jobs in the rural economy to go to the city, where they find no job at all.

The people in my "unemployment" category are full-time learners, earning nothing, entirely supported by their families. My guess is that there are very few poor people in cities in this category and that a more typical situation involves a combination of learning with low-wage jobs, or various kinds of marginal self-employment. A more descriptive model would mix the pure learning of my Sections IV–VI with some employment at low-skill tasks, as in Section VII. Or learning and working at a low-skill job might be tied together, as learning by doing. It would not be hard to set out such hybrid models, but forgoing the convenience of linearity will raise some challenging technical problems. These must be left for future research.

What would an economically *efficient* allocation look like in the external-effect economy of Sections IV–VI? A reasonable conjecture is that a beneficent planner will designate one person to be the technology leader. He will be required to spend full time learning, so his human capital path will be $H(t) = h_0 e^{\delta t}$. Since he has no mass, removing him from production activities entails no cost.[8] Now someone who moves to the city at date s has available to him the technology

$$\frac{\partial h(s,\ t)}{\partial t} = \delta h_0^{\theta} e^{\theta \delta t} h(s,\ t)^{1-\theta}[1 - u(s,\ t)]. \tag{46}$$

I have not worked out the best way to use the technology (46), but one could hardly ask for a better technology to work with!

This example highlights a weakness of my assumption that the ex-

[8] Bob Hall suggested this example in his discussion at the symposium.

ternal benefits of human capital are conferred exclusively by a single "leader." In the symmetric equilibria that I computed, all urban producers are leaders, so the highest skill level is the same as the average. In an equilibrium context, this leads to reasonable allocations, but if one is to gain the ability to use the theory to discover ways to improve on the equilibrium, a better description of the social character of the learning process will be needed. One possibility is to assume an external effect that depends on the average, or perhaps the total, human capital of city producers.

Another possibility is to postulate a local externality under which migrants to the city at date t benefit from the experience of those who arrived earlier.[9] A partial differential equation that expresses this idea is

$$\frac{h_t(s,\ t)}{h(s,\ t)} = \delta\left[1 - \gamma\frac{h_s(s,\ t)}{h(s,\ t)}\right][1 - u(s,\ t)]. \tag{47}$$

In (47), an alternative to (17), the rate of growth of the human capital of a date s migrant is an increasing function of the *difference* between his capital and the capital of those who migrated slightly earlier. I think that these and other ways of reformulating the learning technology offer interesting possibilities for future research.

A useful theory of economic development will necessarily be a theory of transition. The historically observed process begins with the predominantly agricultural society with stable income levels analyzed by Adam Smith and David Ricardo. It moves to the sustained income growth, driven by the expansion of knowledge, that the economically successful societies enjoy today. Rural-urban migration is one element in all such transitions. I hope that the model developed here in the attempt to understand this particular aspect of development may prove helpful in thinking about other aspects as well.

Appendix

Proofs of Propositions 1–3

Proof of Proposition 1

We use the change of variable $Z(t) = \exp\{-[(\delta - \rho)/\sigma]t\}h(t)$ and restate (15) as

$$\frac{dZ(t)}{dt} = aZ(t) - P(t,\ c_0), \tag{A1}$$

[9] See Glaeser (1999) for a suggestive formulation. Equation (47) was proposed to me by Ivar Ekeland.

where

$$a = \frac{\delta\sigma - \delta + \rho}{\sigma}$$

and

$$P(t,\ c_0) = \frac{\delta}{1 - x_0}\left\{c_0 - \exp\left[-\left(\frac{\delta - \rho}{\sigma}t\right)\right]F(x_0)\right\}.$$

We need to show that there is only one value c_0 such that the solution $Z(t)$ of (A1) converges to a constant.

The solution to (A1) is

$$Z(t) = e^{at}\left[h_0 - \int_0^t e^{-as}P(s,\ c_0)ds\right].$$

By the hypothesis $\sigma\delta - \delta + \rho > 0$, $a > 0$, so $z(t)$ converges to a constant if and only if

$$h_0 = \int_0^\infty e^{-as}P(s,\ c_0)ds. \tag{A2}$$

Evidently the right side of (A2) is a well-defined, continuous, strictly increasing function of c_0. At $c_0 = 0$, it is negative. For c_0 sufficiently large, it exceeds h_0. Q.E.D.

Proof of Proposition 2

Assume that the proposition is true on an interval $[0,\ t]$ for some $t \geq 0$. We show that, for some $\epsilon > 0$, this solution can be continued in a unique way on $[t,\ t + \epsilon]$. On such an interval we write

$$H(\tau) = H(t) + \int_t^\tau \left\{\delta H(s) - \frac{\delta}{z(s)}\left[c_0 \exp\left(\frac{\delta - \rho}{\sigma}s\right) - F(x(s))\right]\right\}ds \tag{A3}$$

and

$$z(\tau) = 1 - x(S(\tau)). \tag{A4}$$

Assume first that $S(t) < t$, and for this case choose ϵ so that $S(t + \epsilon) < t$ as well. Then (A4) implies that we can take the function z as well as the function x as fixed on $[t,\ t + \epsilon]$ as we study different possibilities for the function H. Now we view (A3) as defining an operator V on the space C of continuous functions H on $[t,\ t + \epsilon]$ and define a norm on this space by

$$\|H\| = \max_{u \in [t, t+\epsilon]} |H(u)|.$$

Then for any $(H,\ \tilde{H}) \in C$, (A3) implies that

$$\|VH - V\tilde{H}\| \leq \delta \int_t^\tau |H(s) - \tilde{H}(s)|ds \leq \delta\epsilon\|H - \tilde{H}\|.$$

If ϵ is small enough, V is a contraction on C and hence has a unique fixed point in C.

This argument needs modification for the case $t = 0$ since $S(0) = 0$. If no migration ever occurs, so that $x(t) = x_0$ for all t, then $z(t) = 1 - x_0$ for all t, and the argument just given applies to $t = 0$. If $x(t) < x_0$ for any $t > 0$, then

$$\int_0^\infty e^{-\delta t} F'(x(t))dt > \frac{1}{\delta} F'(x_0) = \frac{h_0}{\delta},$$

and the inequality (29) is strict at $t = 0$. This implies that $x'(t) = 0$ for t small enough, and the argument given above again applies. Q.E.D.

Proof of Proposition 3

The proof follows the proof of proposition 1. We again use the change of variable $Z(t) = \exp\{-[(\delta - \rho)/\sigma]t\}H(t)$ and restate (33) as

$$\frac{dZ(t)}{dt} = aZ(t) - Q(t, c_0), \tag{A5}$$

where

$$a = \frac{\delta\sigma - \delta + \rho}{\sigma}$$

and

$$Q(t, c_0) = \frac{\delta}{z(t, c_0)}\left[c_0 - \exp\left(-\frac{\delta - \rho}{\sigma}t\right)F(x(t))\right].$$

Here we write $z(t, c_0)$ to emphasize the fact that catch-up times $S(t, c_0)$ depend on c_0, implying in turn that, for a given migration function x, the additions z to the urban workforce must also depend on c_0.

The solution to (A5) is

$$Z(t) = e^{at}\left[h_0 - \int_0^t e^{-as}Q(s, c_0)ds\right].$$

By the hypothesis $\sigma\delta - \delta + \rho > 0$, $a > 0$, so $z(t)$ converges to a constant if and only if

$$h_0 = \int_0^\infty e^{-as}Q(s, c_0)ds. \tag{A6}$$

The right side of (A6) is negative when $c_0 = 0$ and exceeds h_0 for large enough c_0. Hence there is at least one value of c_0 such that the resulting human capital path satisfies (A5). Q.E.D.

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